Int. J. Heat Mass Transfer. Vol. 5, pp. 1059–1068. Pergamon Press 1962. Printed in Great Britain.

LAMINAR FREE CONVECTION FROM A NON-ISOTHERMAL CONE

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(*Received* 7 *February* 1962 *and in revised form* 23 *April* 1962)

Abstract—Consideration is given to laminar free convection from a vertical non-isothermal right circular cone. Similar solutions for the boundary-layer equations are found to exist when the surface temperature varies as x^n. Numerical solutions of the transformed boundary-layer equations are presented for Prandtl number 0·7, both for the isothermal and linear temperature distributions. The heat-transfer results of previous analyses for the isothermal cone and an experimental correlation for laminar free convection are found to be in excellent agreement with the results reported here.

NOMENCLATURE

a_1, a_2, a_3, a_4, dimensionless constants defined by equations (16–19);
c, dimensionless constant of integration;
$f(x)$, dimensionless function defined by equation (12);
$F(\eta)$, dimensionless dependent variable defined by equation (12);
$g(x)$, dimensionless function defined by equation (11);
$\bar{g}$, acceleration due to gravity, (ft²/h);
G, Grashof number, $\bar{g}\beta L^3 \cos\gamma\,(T - T_\infty)/\nu^2$;
Gr_X, Grashof number based on X, $\bar{g}\beta X^3 \cos\gamma\,(T_w - T_\infty)/\nu^2$;
Gr_L, Grashof number based on L, $\bar{g}\beta L^3 \cos\gamma\,(T_0 - T_\infty)\nu^2$;
Gr_L^*, Grashof number based on L, $\bar{g}\beta L^3 (T_0 - T_\infty)/\nu^2$;
h, local heat-transfer coefficient, $q/(T_w - T_\infty)$(Btu/ft² h degF);
$\bar{h}$, average heat-transfer coefficient, $Q/\pi L^2 \sin\gamma\,(T_0 - T_\infty)$, (Btu/ft² h degF);
k, thermal conductivity of fluid, (Btu/h ft degF);
L, cone slant height, (ft);
n, dimensionless exponent defined by equation (25);
Nu_X, local Nusselt number, hX/k;
Nu_L, average Nusselt number, $\bar{h}L/k$;
Pr, Prandtl number, ν/α;
q, local heat-transfer rate per unit area, (Btu/ft² h);
Q, over-all heat-transfer rate, (Btu/h);
r, dimensionless local cone radius;
R, local cone radius, (ft);
T, temperature, (degF);
u, dimensionless velocity component in X-direction;
U, velocity component in X-direction, (ft/h);
v, dimensionless velocity component in Y-direction;
V, velocity component in Y-direction, (ft/h);
x, dimensionless co-ordinate along a cone ray;
X, co-ordinate along a cone ray, (ft);
y, dimensionless co-ordinate normal to cone surface;
Y, co-ordinate normal to cone surface, (ft).

Greek symbols

α, thermal diffusivity of fluid, (ft²/h);
β, coefficient of thermal expansion of fluid, $-(\partial\rho/\rho\partial T)_p$, (degR⁻¹);
γ, cone apex half-angle;

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η, dimensionless independent variable defined by equation (11);
θ, dimensionless temperature variable, $(T - T_\infty)/(T_w - T_\infty)$;
ρ, density of fluid, (lb/ft^3);
ν, kinematic viscosity of fluid (ft^2/h);
ψ, dimensionless stream function.

Subscripts
∞, ambient conditions;
w, wall conditions;
0, conditions at $X = L$.

INTRODUCTION

RECENTLY, theoretical studies of laminar free convection have received wider attention, especially in cases of non-uniform surface-temperature distributions. Because of the difficulty in solving the boundary-layer equations, relatively few exact solutions exist. However, those which have been reported are derived using the technique of similar solutions. The similarity method is based on the hypothesis that velocity and temperature profiles at two different axial locations differ at most by a co-ordinate-dependent scale factor. By introducing a new independent variable consisting of a combination of the original variables, the boundary-layer equations reduce to a set of ordinary differential equations. Numerical integration of these yields important boundary-layer characteristics. The importance of these solutions is quite evident. In addition to providing results for a specific physical situation and contributing to a better physical understanding of the phenomena, they provide a basis of comparison for approximate boundary-layer techniques. The approximate methods, once verified, can often be used with confidence for situations in which the similarity method is not applicable.

Although numerous authors have investigated laminar free convection for the two-dimensional situation, this paper is concerned primarily with results for axisymmetric flows. Merk and Prins [1] developed the general relations for similar solutions on isothermal axisymmetric forms and showed that the vertical cone has such a solution. Approximate boundary-layer techniques were utilized to arrive at an expression for the dimensionless heat transfer. Braun *et al.* [2] contributed two more isothermal axisymmetric bodies for which similar solutions exist, and used an integral method to provide heat-transfer results for these and the cone over a wide range of Prandtl number. Results obtained by numerically integrating the differential equations with Prandtl number 0·72 were also reported by these investigators.

In this paper, the similar solutions obtainable for free convection from the vertical cone are exhausted. It is shown that similar solutions to the boundary-layer equations for a cone exist when the wall-temperature distribution is a power function of distance along a cone ray. Results obtained by numerically integrating the transformed equations for the isothermal and linear temperature distributions are presented for Prandtl number 0·7.

ANALYSIS

Laminar free convection on a vertical cone is governed by the basic conservation laws: mass, momentum and energy. The boundary-layer form of the equations for steady, axisymmetric, non-dissipative, constant-property flow are

$$\frac{\partial(UR)}{\partial X} + \frac{\partial(VR)}{\partial Y} = 0 \qquad (1)$$

$$U\frac{\partial U}{\partial X} + V\frac{\partial U}{\partial Y} = \nu\frac{\partial^2 U}{\partial Y^2} + \bar{g}\beta\cos\gamma\,(T - T_\infty), \qquad (2)$$

$$U\frac{\partial T}{\partial X} + V\frac{\partial T}{\partial Y} = \alpha\frac{\partial^2 T}{\partial Y^2}. \qquad (3)$$

The co-ordinate system, velocity directions, and gravity orientation are shown in Fig. 1 for the case of T_w higher than T_∞. This is the case to which the present analysis is directed, although it is clear that upon reversal of the direction of gravity the results are applicable when T_w is lower than T_∞.

As pointed out in [2], several simplifications have been incorporated into equations (1–3). Under the assumption that the boundary layer is thin relative to the local cone radius, the local radius to a point in the layer has been replaced with the value at the cone surface, $R(X)$. Evidently, this condition is not satisfied in the neighborhood of the cone tip. Further, since the fluid-density difference, which is the driving force

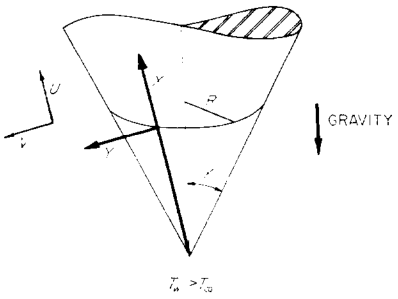

FIG. 1. Physical model and co-ordinates.

for natural convection, has been replaced with the product $\beta(T - T_\infty)$, the equations are limited to small values of this term for liquids but arbitrary values for gases, provided the products of density $\times$ viscosity and density $\times$ conductivity are constant across the layer.

Finally, because the pressure gradient across the boundary layer has been taken as negligible, the equations are strictly applicable to cones of small apex angles.

Complete definition of the problem requires specification of the boundary conditions which are as follows:

$$\left.\begin{aligned} &U = V = 0, \quad T = T_w(X), \quad Y = 0, \\ &U = 0, \qquad\quad T = T_\infty, \qquad Y = \infty. \end{aligned}\right\} \quad (4)$$

Introducing the following dimensionless variables

$$\left.\begin{aligned} &x = \frac{X}{L}, \; y = \frac{Y}{L}, \; r(x) = \frac{R(X)}{L} = x \sin\gamma, \\ &u = \frac{UL}{\nu}, \; v = \frac{VL}{\nu}, \\ &G = \frac{\bar{g}\beta\cos\gamma\,(T - T_\infty)L^3}{\nu^2}, \end{aligned}\right\} \quad (5)$$

where L is the cone slant height, γ the cone apex half-angle and G a Grashof number, equations (1–3) become

$$\frac{\partial(ur)}{\partial x} + \frac{\partial(vr)}{\partial y} = 0, \quad (6)$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{\partial^2 u}{\partial y^2} + G, \quad (7)$$

$$Pr\left(u\frac{\partial G}{\partial x} + v\frac{\partial G}{\partial y}\right) = \frac{\partial^2 G}{\partial y^2}, \quad (8)$$

with boundary conditions

$$\left.\begin{aligned} &u = v = 0, \; G = G_w(x), \quad y = 0, \\ &u = 0, \qquad G = 0, \qquad y = \infty. \end{aligned}\right\} \quad (9)$$

The continuity equation (6) is identically satisfied when a dimensionless stream function ψ is introduced; that is

$$u = \frac{1}{r}\frac{\partial\psi}{\partial y}, \quad v = -\frac{1}{r}\frac{\partial\psi}{\partial x}. \quad (10)$$

If a new independent variable η is defined by

$$\eta = \frac{y}{g(x)} \quad (11)$$

and corresponding dependent variables

$$F(\eta) = \frac{\psi(x, y)}{rf(x)}, \; \theta(\eta) = \frac{G(x, y)}{G_w(x)} = \frac{T - T_\infty}{T_w - T_\infty}, \quad (12)$$

are introduced into equations (7) and (8), the investigation of the existence of similar solutions reduces to determining the functions $f(x)$, $g(x)$ and $G_w(x)$ such that (7) and (8) are two ordinary differential equations for $F(\eta)$ and $\theta(\eta)$. The functions $F'(\eta)$ and $\theta(\eta)$ describe, respectively, the velocity and temperature-difference distributions across the boundary layer. The functions $g(x)$, $f(x)/g(x)$ and G_w represent the growth of boundary-layer thickness, velocity, and temperature difference along the wall.

Substitution of equations (11) and (12) into (7) and (8) yields

$$F''' + a_1 FF'' - a_2 F'^2 + a_3\theta = 0, \quad (13)$$

$$\theta'' + Pr(a_1 F\theta' - a_4 F'\theta) = 0. \quad (14)$$

The boundary conditions transform to

$$\left.\begin{aligned} &F' = F = 0, \quad \theta = 1{\cdot}0, \quad \eta = 0, \\ &F' = 0, \qquad\quad \theta = 0, \qquad \eta = \infty. \end{aligned}\right\} \quad (15)$$

In the above, the prime denotes differentiation with respect to η and a_1 to a_4 are given by

$$a_1 = \frac{g}{x}\frac{\mathrm{d}}{\mathrm{d}x}(fx), \quad (16)$$

$$a_2 = g^2 \frac{d}{dx}\left(\frac{f}{g}\right), \tag{17}$$

$$a_3 = \frac{G_w g^3}{f}, \tag{18}$$

$$a_4 = \frac{fg}{G_w}\frac{dG_w}{dx}. \tag{19}$$

It is apparent that similar solutions are possible when a_1, a_2, a_3 and a_4 are constants. But, since the conditions embodied in equations (16–19) exceed the number of unknowns (f, g, G_w), the constants are not all arbitrary. We note that the functions f and g can be determined from equations (16) and (17) and, with these, equation (18) yields G_w. The functions so determined must then satisfy the condition expressed by (19). Following this procedure, f, g and G_w are found as

$$f = c\left[\left(\frac{2a_1 - a_2}{3}\right)^{a_1} x^{(a_1+a_2)}\right]^{1/(2a_1-a_2)} \tag{20}$$

$$g = \frac{1}{c}\left[\left(\frac{2a_1 - a_2}{3}\right)^{(a_1-a_2)} x^{(a_1-2a_2)}\right]^{1/(2a_1-a_2)} \tag{21}$$

$$G_w = c^4 a_3\left[\left(\frac{2a_1 - a_2}{3}\right)^{(3a_2-2a_1)} x^{(7a_2-2a_1)}\right]^{1/(2a_1-a_2)} \tag{22}$$

provided $2a_1 \neq a_2$. A solution does not exist for $2a_1 = a_2$. The additional parameter c is an integration constant. In the above a_1, a_2, a_3 and c are arbitrary subject to the previously mentioned restriction and a_4 is related to a_1 and a_2 by the relation

$$a_4 = \frac{7a_2 - 2a_1}{3}. \tag{23}$$

As is customary in free-convection analyses we arbitrarily choose a_3 equal to positive unity and let

$$c^4 = \frac{\bar{g}\beta \cos\gamma\,(T_0 - T_\infty)\,L^3}{\nu^2} = Gr_L, \tag{24}$$

where T_0 is the surface temperature at the cone base ($X = L$). For convenience, a new constant n defined by

$$n = \frac{7a_2 - 2a_1}{2a_1 - a_2}, \tag{25}$$

is introduced. Since a_1/a_2 is directly related to n by $a_1/a_2 = (n + 7)/2(n + 1)$ it is convenient to let (arbitrarily)

$$a_1 = \frac{n+7}{4}, \quad a_2 = \frac{n+1}{2}, \tag{26}$$

which satisfies the restriction on a_1 and a_2. According to equation (23) a_4 has the value n. For this selection of constants, equations (20–22) simplify to

$$f = Gr_L^{1/4}\, x^{(3+n)/4} \tag{27}$$

$$g = Gr_L^{-1/4}\, x^{(1-n)/4} \tag{28}$$

$$G_w = Gr_L x^n. \tag{29}$$

It is evident from the last of equations (5), equation (24) and equation (29) that similar solutions are possible for laminar free convection from a non-isothermal cone when the wall-to-environment temperature difference is a power function of the distance from the cone apex; i.e.

$$\frac{T_w - T_\infty}{T_0 - T_\infty} = x^n, \tag{30}$$

where n can be zero or take on any real value. However, it is necessary to restrict n to values not less than zero in order to keep the surface-temperature distribution finite.

The transformation variables now become

$$\left.\begin{aligned} \eta &= Gr_L^{1/4}\, y/x^{(1-n)/4} \\ F(\eta) &= \frac{\psi}{r\, Gr_L^{1/4}\, x^{(3+n)/4}} \\ \theta(\eta) &= \frac{(T - T_\infty)}{(T_0 - T_\infty)x^n} \end{aligned}\right\} \tag{31}$$

while the dimensionless velocities are

$$\left.\begin{aligned} u &= Gr_L^{1/2}\, x^{(1+n)/2}\, F', \\ v &= \tfrac{1}{4}\, Gr_L^{1/4}\, x^{(n-1)/4} \\ &\quad \times [(1-n)\eta F' - (7+n)F]. \end{aligned}\right\} \tag{32}$$

In terms of n, equations (13) and (14) are

$$F''' + \frac{7+n}{4} FF'' - \frac{1+n}{2} F'^2 + \theta = 0, \tag{33}$$

$$\theta'' + Pr\left(\frac{7+n}{4} F\theta' - nF'\theta\right) = 0. \tag{34}$$

Equations (33) and (34) with boundary conditions (15) completely define the mathematical problem of laminar free convection on a vertical cone with a power-function surface-temperature distribution within the framework of the simplifying assumptions. The equations contain the parameters; Prandtl number and n. The parameter n is determined by the specification of the surface-temperature distribution in the form of equation (30). A few of the more important cases obtained by different choices of n are given below.

Isothermal cone ($n = 0$)

The transformed equations for free convection from an isothermal cone are obtained from equations (33) and (34) with $n = 0$. The equations are

$$\left.\begin{aligned} F''' + \tfrac{7}{4} FF'' - \tfrac{1}{2} F'^2 + \theta = 0 \\ \theta'' + \tfrac{7}{4} PrF\theta' = 0 \end{aligned}\right\} \quad (35)$$

while the transformation variable and dimensionless velocities simplify to

$$\left.\begin{aligned} \eta &= Gr_L^{1/4}\, x^{-1/4}\, y \\ u &= Gr_L^{1/2}\, x^{1/2}\, F' \\ v &= \tfrac{1}{4}\, Gr_L^{1/4}\, x^{-1/4} (\eta F' - 7F). \end{aligned}\right\} \quad (36)$$

The isothermal cone was first shown, by Merk and Prins [1], to be a body yielding similar flows. A different choice of constants was selected by these investigators resulting in an analogous set of equations.

Linear surface-temperature distribution ($n = 1$)

For a vertical cone with a linear surface-temperature distribution, equations (33) and (34) reduce to

$$\left.\begin{aligned} F''' + 2FF'' - F'^2 + \theta = 0 \\ \theta'' + Pr(2F\theta' - F'\theta) = 0. \end{aligned}\right\} \quad (37)$$

The similarity variable and velocities are given by

$$\left.\begin{aligned} &\eta = Gr_L^{1/4}\, y \\ &u = Gr_L^{1/2}\, xF', \quad v = -\, 2Gr_L^{1/4}\, F. \end{aligned}\right\} \quad (38)$$

It is interesting to note that for this case the similarity variable is independent of x.

Specified wall heat flux

Other important values of n can be obtained by calculating the local surface heat flux. Utilizing the Fourier–Biot law of heat conduction, the surface flux is given as follows:

$$\begin{aligned} q &= -\, k \left.\frac{\partial T}{\partial Y}\right|_{Y=0} \\ &= -\, \frac{k(T_0 - T_\infty)}{L}\, Gr_L^{1/4}\, x^{(5n-1)/4}\, \theta'(0). \end{aligned} \quad (39)$$

From the above it is evident that different surface flux distributions can be obtained by specifying the value of $(5n - 1)/4$. For example, choosing $n = 1/5$ gives a constant heat flux surface condition for which the transformed equations are

$$\left.\begin{aligned} F''' + \tfrac{9}{5} FF'' - \tfrac{3}{5} F'^2 + \theta = 0 \\ \theta'' + Pr(\tfrac{9}{5} F\theta' - \tfrac{1}{5} F'\theta) = 0. \end{aligned}\right\} \quad (40)$$

The similarity variable and the dimensionless velocities for the case of constant surface heat flux are

$$\left.\begin{aligned} \eta &= Gr_L^{1/4}\, x^{-1/5}\, y \\ u &= Gr_L^{1/2}\, x^{3/5}\, F' \\ v &= \tfrac{1}{5}\, Gr_L^{1/4}\, x^{-1/5} (\eta F' - 9F). \end{aligned}\right\} \quad (41)$$

Sparrow and Gregg [3] have dealt with the case of uniform surface heat flux for the vertical flat plate and have shown that the resulting surface temperature also varies with the one-fifth power of the distance from the leading edge.

SOLUTION OF EQUATIONS

The reduced equations for the isothermal cone (35) and the equations for the cone with a linear surface-temperature distribution (37) have been solved for Prandtl number 0·7 using numerical techniques. The numerical method, fourth-order Runge-Kutta forward integration, requires that at the starting point of the integration the function and its first two derivatives be specified for a third-order equation; whereas, for a second-order equation, the function and its first derivative must be prescribed. As is seen from the boundary conditions of equation (15), $F''(0)$ and $\theta'(0)$ are not known. Thus, the computational problem reduces to a systematic search for the values of these derivatives which lead to solutions of the equations satisfying the end conditions $F'(\infty) = 0$ and $\theta(\infty) = 0$. The details of the integration formulas and an iterative technique for determining the unknown initial conditions are described in [4].

The initial value results of the numerical integrations are believed to be correct to at least five digits. In both cases, the integration was not considered satisfactory until the initial values $F''(0)$ and $\theta'(0)$ did not change in at least the sixth digit upon correction by the previously mentioned technique. This fact, plus (a) optimization of the integration interval size, (b) a check of previously obtained results for the rotating disk [5], and (c) the estimated errors inherent in the integration method, insures that five digits is a conservative estimate of the accuracy of the initial values $F''(0)$ and $\theta'(0)$. Numerical calculations were performed on a Datatron 204 digital computer. Short tables of the functions F, F', F'', θ, and θ' are given in the Appendix.

RESULTS

Local heat transfer

The most important result of practical interest is the heat transfer. It is customary to express heat-transfer results in terms of heat-transfer coefficients and Nusselt numbers. A local heat-transfer coefficient and local Nusselt number may be defined as follows

$$h \equiv \frac{q}{T_w - T_\infty}, \qquad Nu_X = \frac{hX}{k}. \tag{42}$$

Utilizing the previous result for the local heat flux (39) we find

$$Nu_X = - Gr_X^{1/4}\, \theta'(0), \tag{43}$$

where Gr_X is a Grashof number based on length X and the local wall-to-environment temperature difference $(T_w - T_\infty)$.

Using the integration results for Prandtl number 0·7, the dimensionless heat transfer for the isothermal cone is

$$Nu_X = 0{\cdot}45110\, Gr_X^{1/4}, \tag{44}$$

while for a linear surface-temperature distribution we obtain

$$Nu_X = 0{\cdot}56699\, Gr_X^{1/4}. \tag{45}$$

Merk and Prins [1] integrated the equations for the isothermal vertical cone using an integral method. Their result when evaluated at Prandtl number 0·7 is in excellent agreement with that found here. Their coefficient in equation (44) was lower by less than 1·6 per cent. The numerical integration result of [2] for Prandtl number 0·72 is less than 1·2 per cent greater than that found here when their results are transformed to the definitions of this paper. Braun *et al.* [2] also noted that, although the method of Saunders used by Merk and Prins gave good agreement for Prandtl number of order unity, it was not reliable at extremes of Prandtl number.

Over-all heat transfer

The over-all heat transfer Q is obtained by integrating equation (39) over the cone lateral surface area. So

$$Q = 2\pi L^2 \sin\gamma \int_0^1 qx\, \mathrm{d}x, \tag{46}$$

where L is the cone slant height. Introducing the following definitions for a mean heat-transfer coefficient and Nusselt number:

$$\bar{h} = \frac{Q}{\pi L^2 \sin\gamma\,(T_0 - T_\infty)}, \quad \overline{Nu}_L = \frac{\bar{h}L}{k}, \tag{47}$$

gives the dimensionless heat-transfer result

$$\overline{Nu}_L = -\left(\frac{8}{5n + 7}\right) Gr_L^{1/4}\, \theta'(0). \tag{48}$$

Upon substituting the appropriate integration results for Prandtl number 0·7,

$$\overline{Nu}_L = 0{\cdot}51554\, Gr_L^{1/4}, \tag{49}$$

for the isothermal cone and

$$\overline{Nu}_L = 0{\cdot}37799\, Gr_L^{1/4} \tag{50}$$

for the cone with a linear wall-temperature distribution.

Again the results of [1] show excellent agreement with a coefficient less than 1·3 per cent lower than that of equation (49). Furthermore, Jakob and Linke [6] combined the experimental results of numerous authors for vertical cylinders, vertical planes, horizontal wires, spheres, and blocks in air, water, alcohol and oil, and correlated these results for laminar flow with the following equation:

$$\bar{N}u_L = 0{\cdot}555\,(Gr_L^*\,Pr)^{1/4}$$

where $Gr_L^* = \bar{g}\beta(T_0 - T_\infty)L^3/\nu$. If the gravitational acceleration in the above relation is interpreted as the component parallel to the body surface, evaluation of Jakob and Linke's expression for Prandtl number 0·7 results in a numerical coefficient also 1·3 per cent less than that of equation (49).

Also of interest is that the heat-transfer results, both local and over-all, for the cone with the linear surface-temperature distribution deviate considerably from those for the isothermal cone. In fact, the isothermal-cone results were 20 per cent lower and 36 per cent higher than the respective local and over-all Nusselt numbers for the non-isothermal cone. Hence, application of isothermal heat-transfer relationships in the presence of non-isothermal temperature distributions can result in considerable error in heat-transfer calculations.

Velocity and temperature profiles

The tangential velocity component and temperature profiles for both the isothermal and linear-surface-temperature cases are shown in Figs. 2 and 3. The profiles exhibit the usual free-convection shapes. Note that the dimensionless tangential-flow function for the isothermal cone attains a maximum 22 per cent greater than that for the cone with a linear wall-temperature distribution.

CONCLUSION

It has been shown that similar solutions to the laminar boundary-layer equations exist for free convection from a vertical cone when the surface-temperature distribution is a power function of the distance from the cone apex. Numerical solutions of the transformed differential equations have been presented for the isothermal and linear surface-temperature distributions with Prandtl number 0·7. The approximate heat-transfer results of Merk and Prins for the isothermal cone when evaluated for Prandtl number 0·7 were in excellent agreement with those found here. The experimental correlation obtained by Jakob and Linke for the mean Nusselt number in laminar isothermal free convection when evaluated on the basis of the effective gravitational acceleration and Prandlt number 0·7 gave results within 1·3 per cent of

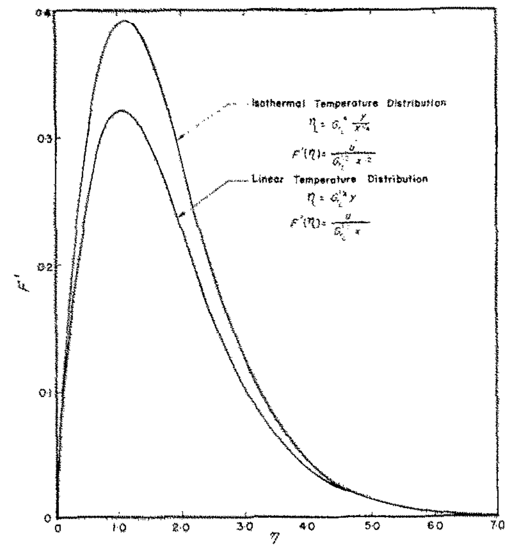

FIG. 2. Dimensionless tangential flow-function profiles ($Pr = 0{\cdot}7$).

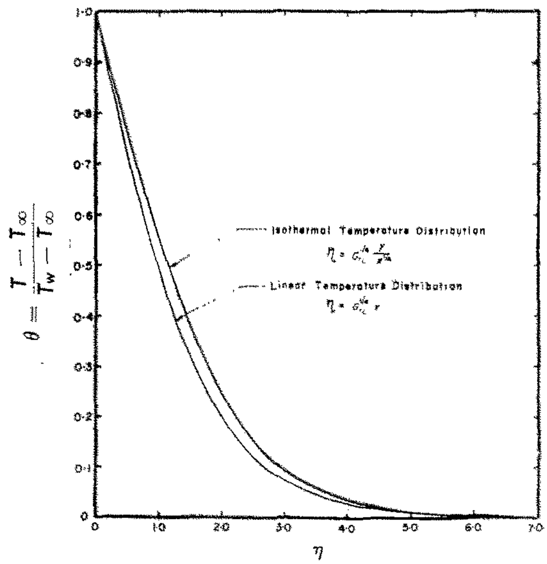

FIG. 3. Dimensionless temperature profiles ($Pr = 0{\cdot}7$).

the result obtained herein for the isothermal cone.

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APPENDIX

Integration results for the functions, F, F', F'', θ and θ' are given in Tables 1 and 2 below.† The function values for the isothermal cone are given in terms of a slightly different argument than that used in this paper. The original integrations were performed using the argument $\bar{\eta} = (1/\sqrt{2})\eta$ for the isothermal case.

Table 1. Tabulation of function values for isothermal temperature distribution

$\bar{\eta} = (1/\sqrt{2})\eta$	F	F'	$-F''$	θ	$-\theta'$
0·0	0·00000	0·00000	− 0·81959	1·00000	0·45110
0·2	0·29127 (− 1)	0·19343	− 0·55366	0·87252	0·44953
0·4	0·10277	0·31637	− 0·32167	0·74649	0·43982
0·6	0·20241	0·37916	− 0·12965	0·62494	0·41738
0·8	0·31274	0·39415	0·15411 (− 1)	0·51163	0·38179
1·0	0·42217	0·37508	0·11154	0·40992	0·33610
1·2	0·52297	0·33524	0·16350	0·32199	0·28524
1·4	0·61092	0·28584	0·18098	0·24857	0·23356
1·6	0·68454	0·23504	0·17528	0·18910	0·18170
1·8	0·74423	0·18791	0·15655	0·14215	0·14602
2·0	0·79143	0·14698	0·13242	0·10584	0·11187
2·2	0·82804	0·11304	0·10778	0·78225 (− 1)	0·84482 (− 1)
2·4	0·85601	0·85803 (− 1)	0·85302 (− 1)	0·57486 (− 1)	0·63091 (− 1)
2·6	0·87713	0·64471 (− 1)	0·66125 (− 1)	0·42062 (− 1)	0·46719 (− 1)
2·8	0·89294	0·48063 (− 1)	0·50471 (− 1)	0·30678 (− 1)	0·34376 (− 1)
3·0	0·90469	0·35613 (− 1)	0·38075 (− 1)	0·22321 (− 1)	0·25175 (− 1)
3·2	0·91338	0·26262 (− 1)	0·28469 (− 1)	0·16211 (− 1)	0·18372 (− 1)
3·4	0·91977	0·19294 (− 1)	0·21141 (− 1)	0·11758 (− 1)	0·13372 (− 1)
3·6	0·92446	0·14133 (− 1)	0·15617 (− 1)	0·85206 (− 2)	0·97146 (− 2)
3·8	0·92789	0·10328 (− 1)	0·11489 (− 1)	0·61699 (− 2)	0·70476 (− 2)
4·0	0·93040	0·75333 (− 2)	0·84251 (− 2)	0·44655 (− 2)	0·51076 (− 2)
4·5	0·93408	0·34026 (− 2)	0·38407 (− 2)	0·19871 (− 2)	0·22774 (− 2)
5·0	0·93573	0·15280 (− 2)	0·17344 (− 2)	0·88328 (− 3)	0·10132 (− 2)
5·5	0·93648	0·68373 (− 3)	0·77880 (− 3)	0·39243 (− 3)	0·45032 (− 3)
6·0	0·93681	0·30528 (− 3)	0·34845 (− 3)	0·17432 (− 3)	0·20006 (− 3)
7·0	0·93702	0·60687 (− 4)	0·69323 (− 4)	0·34394 (− 4)	0·39466 (− 4)
8·0	0·93706	0·12115 (− 4)	0·13735 (− 4)	0·67932 (− 5)	0·77842 (− 5)
10·0	0·93707	0·58778 (− 6)	0·54297 (− 6)	0·27580 (− 6)	0·30280 (− 6)

† The values given in the tables must be multiplied by the power of ten given in parentheses.

Table 2. Tabulation of function values for linear temperature distribution

η	F	F'	$-F''$	θ	$-\theta'$
0·0	0·0	0·0	− 0·72480	1·0	0·56699
0·2	0·13201 (− 1)	0·12572	− 0·53619	0·88723	0·55773
0·4	0·47926 (− 1)	0·21603	− 0·37087	0·77793	0·53319
0·6	0·97567 (− 1)	0·27568	− 0·22978	0·67469	0·49771
0·8	0·15648	0·30960	− 0·11354	0·57932	0·45510
1·0	0·22002	0·32274	− 0·21917 (− 1)	0·49290	0·40862
1·2	0·28452	0·31992	0·46440 (− 1)	0·41594	0·36102
1·4	0·34722	0·30556	0·93882 (− 1)	0·34843	0·31446
1·6	0·40623	0·28355	0·12350	0·28998	0·27050
1·8	0·46035	0·25712	0·13875	0·23998	0·23016
2·0	0·50895	0·22878	0·14307	0·19764	0·19400
2·2	0·55186	0·20041	0·13960	0·16209	0·16221
2·4	0·58920	0·17328	0·13101	0·13247	0·13471
2·6	0·62131	0·19821	0·11944	0·10794	0·11123
2·8	0·64865	0·12560	0·10648	0·87734 (− 1)	0·91398 (− 1)
3·0	0·67173	0·10563	0·93273 (− 1)	0·71165 (− 1)	0·74801 (− 1)
3·2	0·69107	0·88258 (− 1)	0·80556 (− 1)	0·57627 (− 1)	0·61015 (− 1)
3·4	0·70720	0·73343 (− 1)	0·68780 (− 1)	0·46599 (− 1)	0·49634 (− 1)
3·6	0·72056	0·60667 (− 1)	0·58179 (− 1)	0·37638 (− 1)	0·40284 (− 1)
3·8	0·73160	0·49987 (− 1)	0·48834 (− 1)	0·30372 (− 1)	0·32636 (− 1)
4·0	0·74067	0·41051 (− 1)	0·40729 (− 1)	0·24489 (− 1)	0·26400 (− 1)
4·5	0·75682	0·24804 (− 1)	0·25328 (− 1)	0·14262 (− 1)	0·15461 (− 1)
5·0	0·76651	0·14809 (− 1)	0·15413 (− 1)	0·82862 (− 2)	0·90123 (− 2)
5·5	0·77228	0·87700 (− 2)	0·92459 (− 2)	0·48076 (− 2)	0·52390 (− 2)
6·0	0·77569	0·51634 (− 2)	0·54931 (− 2)	0·27871 (− 2)	0·30406 (− 2)
7·0	0·77886	0·17706 (− 2)	0·19041 (− 2)	0·93560 (− 3)	0·10217 (− 2)
8·0	0·77994	0·60220 (− 3)	0·65080 (− 3)	0·31389 (− 3)	0·34282 (− 3)
10·0	0·78043	0·69534 (− 4)	0·74570 (− 4)	0·35379 (− 4)	0·38557 (− 4)

Résumé—On fait ici une étude de la convection libre laminaire autour d'un cone, à base circulaire, non isotherme placé verticalement. On trouve qu'il existe des solutions semblables pour les équations de couche limite quand la température de surface varie comme x^n. Des solutions numériques des équations de couche limite transformées sont données pour un nombre de Prandtl égal à 0,7, dans le cas de distributions de température isotherme et linéaire.

Les résultats rapportés ici sont en excellent accord avec ceux des études antérieures sur les échanges thermiques à partir d'un cône isotherme et les résultats expérimentaux de convection libre correspondants.

Zusammenfassung—Es wird die laminare freie Konvektion an einem nichtisolierten senkrecht stehenden Kreiskegel untersucht. Für die Grenzschichtgleichungen ergeben sich Ähnlichkeitslösungen, wenn sich die Oberflächentemperatur nach x^n ändert. Numerische Lösungen der transformierten Grenzschichtgleichung sind sowohl bei isothermer als auch bei linearer Temperaturverteilung für die Prandtl-Zahl 0,7 angegeben. Die Ergebnisse früherer Untersuchungen des isothermen Kegels und eine experimentelle Korrelation für laminare freie Konvektion stimmen sehr gut mit den hier ermittelten Werten überein.

Аннотация—Рассматривается ламинарная свободная конвекция при теплообмене вертикально неизотермического прямого кругового конуса. Найдено, что при изменении температуры поверхности, пропорциональной x^n, существуют подобные решения уравнений пограничного слоя. Численные решения преобразованных уравнений пограничного слоя представлены для числа Прандтля равного 0,7, как для изотермических, так и линейных распределений температуры. Результаты теплообмена для изотермического конуса, найденные на основе предыдущих анализов, и экспериментальное соотношение для ламинарной свободной конвекции хорошо согласуются с результатами, представленными в данной работе.